Modelling Spinning Dust Emissions

Zheng Zhang

Jodrell Bank Centre for Astrophysics, Department of Physics
University of Manchester
Manchester, UK

JBCA Colloquium, Manchester, 28 May 2025

OUTLINE

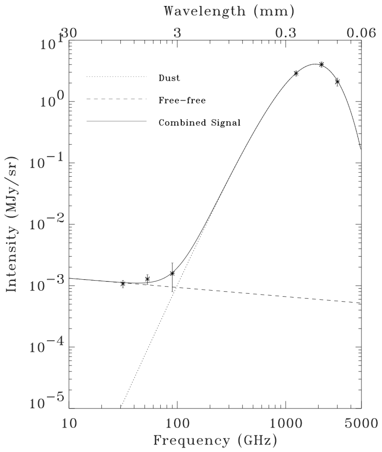
Anomalous Microwave Emission (AME)

Background

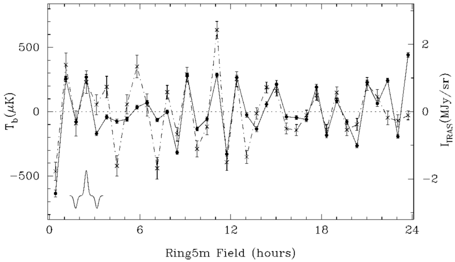
Early Clues: COBE

- Anomalous components were noted in the 10-60 GHz range.
- Inconsistent SED with free-free, synchrotron, or thermal dust.
- But *spatially* correlated with dust.

Kogut et al. (1996) ApJ, 464, L5



Anomalous Microwave Emission (AME)
Background



Late 1990s: OVRO

- Found strong correlation between 14.5 GHz emission and far-IR dust.
- But with a spectrum not matching known foregrounds.
- The term “Anomalous Microwave Emission” began to be used.

Leitch et al. (1997) ApJ, 486, L23

Anomalous Microwave Emission (AME)

Background

Early Clues: COBE

- Anomalous components were noted in the 10-60 GHz range.
- The unexplained emission is *spatially* correlated with dust.
- Inconsistent SED with free-free, synchrotron, or thermal dust.

Kogut et al. (1996) ApJ, 464, L5

Late 1990s: OVRO

- Found strong correlation between 14.5 GHz emission and far-IR dust.
- But with a spectrum not matching known foregrounds.
- The term “Anomalous Microwave Emission” began to be used.

Leitch et al. (1997) ApJ, 486, L23

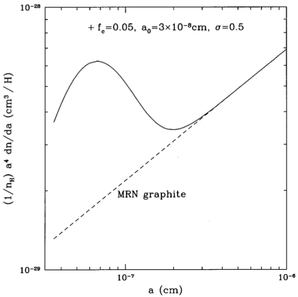
AME was identified as a distinct component.

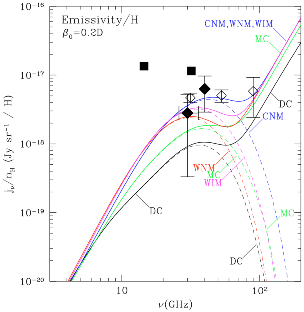
Anomalous Microwave Emission (AME)
Spinning Dust Hypothesis

- Spherical grain radiation: $P \propto \frac{4}{9c^3}\mu^2\omega^4$
- Steady state condition, $\frac{d}{dt}\left(\frac{I\langle\omega^2\rangle}{2}\right) = 0$, gives

$$0 = \frac{3kT}{\tau_H}G - \frac{I\langle\omega^2\rangle}{\tau_H}F - \frac{4\mu^2\langle\omega^4\rangle}{9c^3},$$

  which provides an estimate for $\langle\omega^2\rangle$.
- Assuming ω to follow a Boltzmann distribution, the emissivity is





$$\frac{j_\nu}{n_{\rm H}} = \left(\frac{8}{3\pi}\right)^{1/2}\frac{1}{n_{\rm H}c^3}\int da\,\frac{dn}{da}\,\frac{2}{3}\,\frac{\mu^2\omega^6}{\langle\omega^2\rangle^{3/2}}\exp\left\{\frac{-3\omega^2}{2\langle\omega^2\rangle}\right\}$$

***Electric Dipole Radiation from Spinning Dust Grains**, Draine & Lazarian, ApJ, 508, 157 (1998)*

Anomalous Microwave Emission (AME)

Strengthening the Case

- WMAP full-sky maps highlighted AME as a distinct foreground.
- Modelled Galactic foregrounds: noted emission correlated with dust but inconsistent with thermal dust or synchrotron.

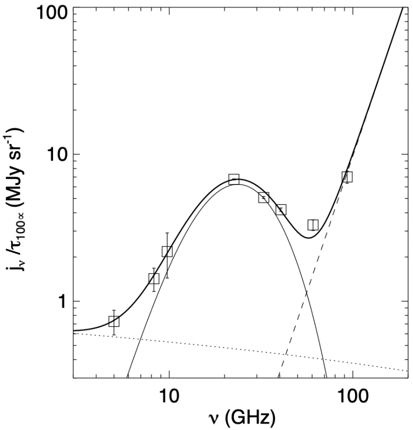
  Bennett et al. (2003) ApJS, 148, 97

- Fitted multifrequency data using spinning dust models.

  Finkbeiner (2004) ApJ, 614, 186

Spectral Consistency with the Model
Spatial Correlation with Infrared Emission

Fitted Green Bank + WMAP



Anomalous Microwave Emission (AME)

Strengthening the Case

- WMAP full-sky maps highlighted AME as a distinct foreground.
- Modelled Galactic foregrounds: noted emission correlated with dust but inconsistent with thermal dust or synchrotron.

Bennett et al. (2003) ApJS, 148, 97

- Fitted multifrequency data using spinning dust models.

Finkbeiner (2004) ApJ, 614, 186

The Planck Era

- Provided precise, broad-frequency maps of the sky.
- Conducted detailed AME studies across Galactic regions
- Confirmed the spinning dust explanation

Planck Collaboration Int. XV (2014) A&A, 565, A103

Spectral Consistency with the Model
Spatial Correlation with Infrared Emission

Environmental Associations

Anomalous Microwave Emission (AME)

Refinement and Further Investigations

Refined Spinning Dust Models:

- SpDust: A Fokker-Planck approach to calculate the angular velocity distribution.
  Ali-Haïmoud et al. (2009) MNRAS, 395, 1055
- Langevin equation method: accounting for the wobbling motion of disk-like grains and the transient spin-up due to ion collisions.
  T. Hoang, et al. (2010) ApJ, 715, 1462
- SpDust2: extended spdust by modelling the rotation of wobbling.
  Silsbee et al. (2011) MNRAS, 411, 2750
- Effects of irregular grain shape, transient heating
  T. Hoang, et al. (2011) ApJ, 741, 87

Investigations into

- Polarization of AME (expected to be very weak).
- AME in extragalactic sources.
- Non-PAH candidates (e.g., nanosilicates).
- Environmental variability and diagnostics.

See Dickinson et al. (2018) for a comprehensive review.

Spinning Dust Emissions
What's left to uncover?



Spinning Dust Emissions and AME... Still spinning

Pain point in AME modelling and fitting

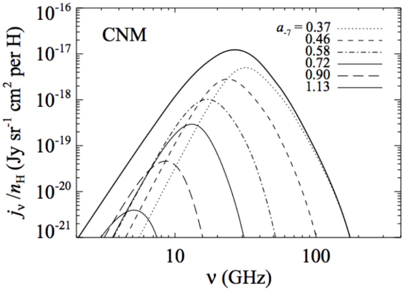
Ad-hoc Modelling:

- Specific grain shapes
- Log-normal size distribution
- Limited statistical mechanics (e.g., given internal fluctuations)

A Wealth of Parameters:

- **Grain size distribution:** $\sim$ 5 params
- **ISM environment:** $\sim$ 10 params

But simple spectral behaviour



Modeling Spinning Dust Emission

Reframing the Problem

Where do we go from here?

1. Generalize physically-motivated parts to broader contexts
2. Relax assumptions in ad-hoc parts
3. Improve accuracy in rotational statistics
4. Develop modular and portable code
5. Reduce the dimensionality of the parameter space

Our approach so far

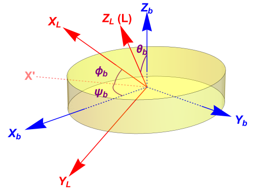
- SpyDust
  $\rightarrow$ Points 1 & 4
- *Moment Expansion*
  $\rightarrow$ Points 2 & 5 (In prep.)
- *Improved Fokker-Planck*
  $\rightarrow$ Point 3 (In prep.)

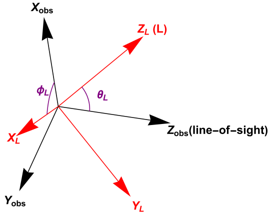
Our framework is full-Stokes and supports various rotational statistics.

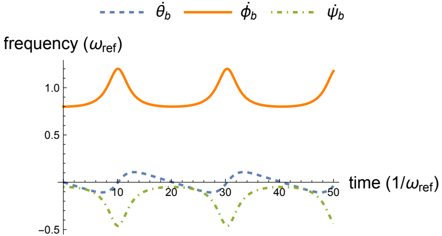
As a fitting tool, we aim for high dynamic range with reduced dimensionality.

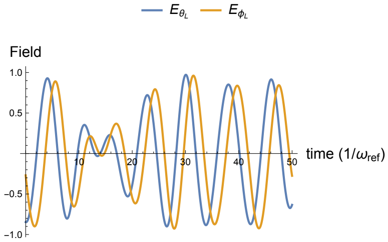
SPYDUST: an extended implementation

Grain Dynamics









$$\dot{\theta}_b = \left(\frac{1}{I_1} - \frac{1}{I_2}\right) L \sin\theta_b \sin\psi_b \cos\psi_b,$$

$$\dot{\phi}_b = \left(\frac{\sin^2\psi_b}{I_1} + \frac{\cos^2\psi_b}{I_2}\right) L,$$

$$\dot{\psi}_b = \left(\frac{1}{I_3} - \frac{\sin^2\psi_b}{I_1} - \frac{\cos^2\psi_b}{I_2}\right) L \cos\theta_b.$$

SpyDust: Single Grain Directional Radiation
Stokes I

It proves to be useful if we reparameterise the principal moments of inertia as below

$$\frac{1}{I_1} = \frac{1+\alpha}{I_{\rm ref}}, \qquad \frac{1}{I_2} = \frac{1-\alpha}{I_{\rm ref}}, \qquad \frac{1}{I_3} = \frac{1+\beta}{I_{\rm ref}},$$

where we have required $\frac{1}{I_1} - \frac{1}{I_2} = \min\left\{ \frac{1}{I_j} - \frac{1}{I_k} \,\middle|\, \text{for all } j,k \text{ satisfying } I_j < I_k \right\}$.
Assuming that $\alpha \approx 0$, then the radiative electric field is approximately four normal modes:

$$\omega^{(1)} = \Omega, \qquad P_\omega^{(1)} = \frac{1}{2}\mu_\parallel^2\omega^4\left[3+\cos(2\theta_L)\right]\sin^2\theta_b,$$

$$\omega^{(2)} = \Omega\left|1+\beta\cos\theta_b\right|, \qquad P_\omega^{(2)} = \frac{1}{2}\mu_\perp^2\omega^4\left[3+\cos(2\theta_L)\right]\cos^4\left(\frac{\theta_b}{2}\right),$$

$$\omega^{(3)} = \Omega\left|1-\beta\cos\theta_b\right|, \qquad P_\omega^{(3)} = \frac{1}{2}\mu_\perp^2\omega^4\left[3+\cos(2\theta_L)\right]\sin^4\left(\frac{\theta_b}{2}\right),$$

$$\omega^{(4)} = \Omega\left|\beta\cos\theta_b\right|, \qquad P_\omega^{(4)} = \mu_\perp^2\omega^4\sin^2\theta_L\sin^2\theta_b,$$

SpyDust: Single Grain Directional Radiation
Polarisation

It proves to be useful if we reparameterise the principal moments of inertia as below

$$\frac{1}{I_1} = \frac{1+\alpha}{I_{\rm ref}}, \qquad \frac{1}{I_2} = \frac{1-\alpha}{I_{\rm ref}}, \qquad \frac{1}{I_3} = \frac{1+\beta}{I_{\rm ref}},$$

where we have required $\frac{1}{I_1} - \frac{1}{I_2} = \min\left\{ \frac{1}{I_j} - \frac{1}{I_k} \,\middle|\, \text{for all } j, k \text{ satisfying } I_j < I_k \right\}$.

Full Stokes components perceived by the observer

$$P_S^{(m)} = \mu^2 \omega^4 \mathcal{A}_m(u)\, \mathcal{B}_m(\theta_b)\, \mathcal{C}_S^{(m)}(\theta_L, \phi_L)$$

where $S = I, Q, U, V$, and $u = \mu_\perp^2/\mu^2$.

Toward the Overall SED

Hierarchical Ensemble Averaging

The total spectral energy density (SED) can be obtained as:

$$\bar{\mathcal{P}}_S(\omega) = \sum_{m=1}^{4} \int \mathrm{d}\Omega \mathrm{d}\theta_b \mathrm{d}\theta_L \mathrm{d}\phi_L \mathrm{d}\mu \mathrm{d}u \mathrm{d}\beta \; \delta_D\left(\omega - \omega^{(m)}(\Omega, \beta, \theta_b)\right)$$

$$\times P_S^{(m)}(\omega, \theta_b, \theta_L, \phi_L, \mu, u)\, f(\Omega, \theta_b, \theta_L, \phi_L, \mu, u, \beta)$$

Characteristic timescales determine the core idea:

- **Time scales:** $1/\nu_{\rm obs} \ll$ time of internal (θ_b) transitions $\ll$ time of L transport.
- **Ansatz:**

$$f(\Omega_m, \theta_b, \theta_L, \phi_L, \mu, u, \beta, \mathrm{ENV}) =$$

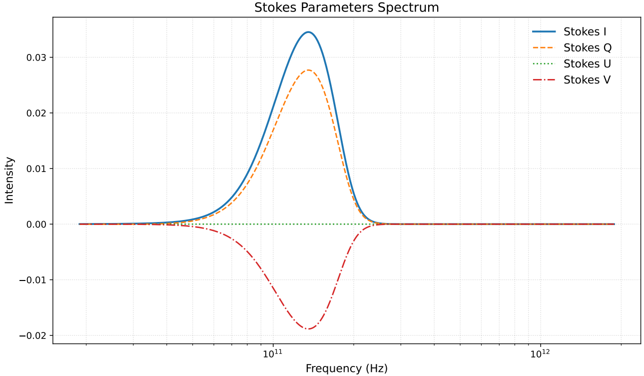
$$f(\Omega_m|\mu, u, \beta, T_d, \vec{\boldsymbol{p}}_1)\, f(\theta_b|T_d)\, f(\theta_L, \phi_L|\vec{\boldsymbol{p}}_2)\, f(\mu, u, \beta, T_d, \vec{\boldsymbol{p}}_1, \vec{\boldsymbol{p}}_2)$$

Toward the Overall SED

An example with polarisation

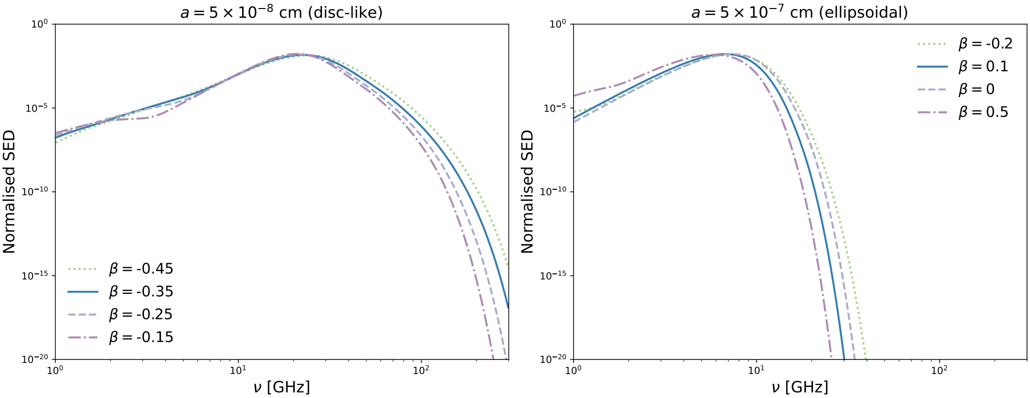
Anisotropic external alignment:

$f(\cos\theta_L, \phi_L)$ is concentrated at $(0.3, \pi/2)$, with the standard deviation 0.1.



Characteristic timescales determine the core idea:

SpyDust: Extensions and Improvements

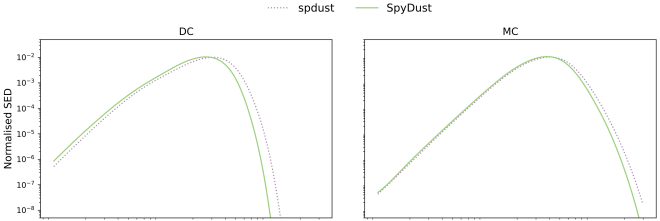


Zhang & Chluba JCAP03(2025)038

SpyDust: Extensions and Improvements

spdust vs spydust

- SpDust formulates single-grain emission as that of a perfect 2D disk.
- Even when the rotational statistics, environment and grain assumptions remain the same, SpyDust still makes a difference when only the mapping between rotation and observation frequency is corrected.



Zhang & Chluba JCAP03(2025)038

Perturbative Representation with Derivative Spectra

Formalism of moment expansion

Abstractly, the total emission can be written in the following form:

$$\bar{\mathcal{P}}_S(\nu) = \int \mathrm{d}\boldsymbol{p}\, \mathcal{P}_S(\nu, \boldsymbol{p}) f(\boldsymbol{p}),$$

where $\boldsymbol{p}$ represents the parameter vector and $\mathcal{P}_S$ denotes the integrand with the distribution $f(\boldsymbol{p})$ factored out.
Taylor expanding $\mathcal{P}_S$ with respect to $\boldsymbol{p}$ around a pivot value $\bar{\boldsymbol{p}}$:

$$\bar{\mathcal{P}}_S(\nu) = \sum_{n_1=0}^{\infty} \cdots \sum_{n_s=0}^{\infty} \frac{\mathcal{W}_{[n_1,\ldots,n_s]}}{\prod_{i=1}^{s} n_i!} \Theta_{[n_1,\ldots,n_s]}(\nu).$$

where $\mathcal{W}_{[n_1,\ldots,n_s]} \equiv \langle (p_1 - \bar{p}_1)^{n_1} \ldots (p_s - \bar{p}_s)^{n_s} \rangle, \quad \Theta_{[n_1,\ldots,n_s]} \equiv \partial_{p_1}^{n_1} \ldots \partial_{p_s}^{n_s} \mathcal{P}_S \big|_{\bar{\boldsymbol{p}}}.$

Derivative Spectral Base for Spinning Dust

Construction

The overall SED can now be rewritten as

$$\bar{\mathcal{P}}_S(\omega) = \omega^4 \int \mathrm{d}\,\mathrm{ENV}\mathrm{d}\mu\mathrm{d}u\mathrm{d}\beta$$

$$\times \left[\sum_{m=1}^{4} \mu^2 \mathcal{A}_m(u)\,\mathcal{R}_m(\omega, \mu, u, \beta, T_d, \vec{\boldsymbol{p}}_1)\,\bar{\mathcal{C}}_S^{(m)}(\vec{\boldsymbol{p}}_2)\right] f(\mu, u, \beta, T_d, \vec{\boldsymbol{p}}_1, \vec{\boldsymbol{p}}_2),$$

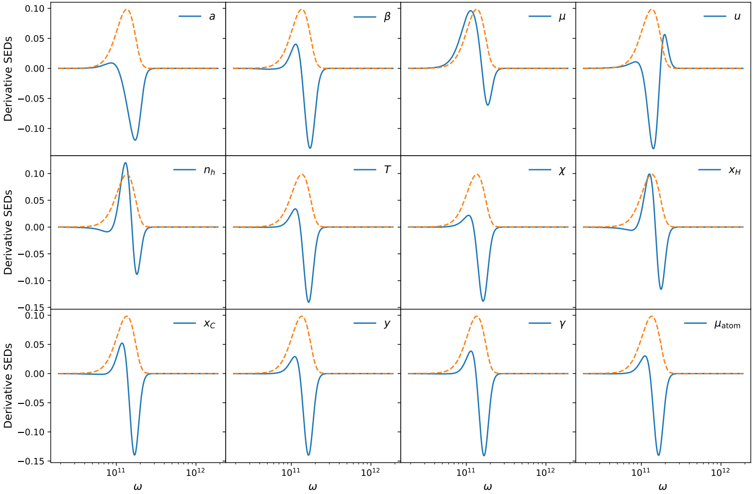
where we have defined

$$\mathcal{R}_m(\omega, \mu, u, \beta, T_d, \vec{\boldsymbol{p}}_1) = \int \mathrm{d}\theta_b\,\mathcal{B}_m(\theta_b)\partial_\omega\Omega_m(\omega, \theta_b, \beta)\, f(\theta_b|T_d) f(\Omega_m|\mu, u, \beta, T_d, \vec{\boldsymbol{p}}_1),$$

$$\bar{\mathcal{C}}_S^{(m)}(\vec{\boldsymbol{p}}_2) = \int \mathrm{d}\theta_L\mathrm{d}\phi_L\,\mathcal{C}_S^{(m)}(\theta_L, \phi_L) f(\theta_L, \phi_L|\vec{\boldsymbol{p}}_2).$$

Derivative Spectral Base for Spinning Dust

An example with CNM



Derivative Spectral Base for Spinning Dust

Dimensionality reduction via PCA

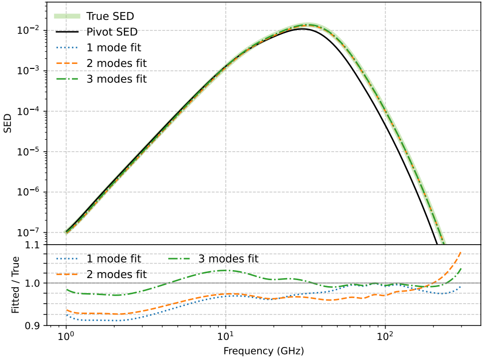
- Covariance between parameters:

$$\mathrm{cov}(p_1, p_2) \equiv \frac{\langle \partial_{p_1} \ln I_\nu, \partial_{p_2} \ln I_\nu \rangle}{|\partial_{p_1} \ln I_\nu||\partial_{p_2} \ln I_\nu|}$$

where

$$\left\langle \frac{\partial \ln I_\nu}{\partial p_1}, \frac{\partial \ln I_\nu}{\partial p_2} \right\rangle = \int \frac{\partial \ln I_\nu(p_1)}{\partial_{p_1}} \frac{\partial \ln I_\nu(p_2)}{\partial_{p_2}} \, d\nu,$$

$$|\partial_{p_1} \ln I_\nu| \equiv \sqrt{\langle \partial_{p_1} \ln I_\nu, \partial_{p_1} \ln I_\nu \rangle}.$$



Zhang & Chluba JCAP03(2025)038

Rotation Statistics: Diffusion Approximation

$$\partial_t P(x,t) = \int \left[w(x|x')P(x',t) - w(x'|x)P(x,t)\right] dx'$$

$$\Longrightarrow \partial_t P(x,t) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \frac{\partial^n}{\partial x^n} \left(a_n(x)P(x,t)\right), \text{ where } a_n = \int \xi^n w(x+\xi|x)\, d\xi.$$

Truncating at Second Order

- Fokker–Planck Equation:

$$\partial_t P(x,t) = -\,\partial_x \left[A(x)P(x,t)\right] + \frac{1}{2}\partial_x^2 \left[B(x)P(x,t)\right]$$

- Applications
  - $A, B \Rightarrow P$
  - $A, P \Rightarrow B$
  - $B, P \Rightarrow A$

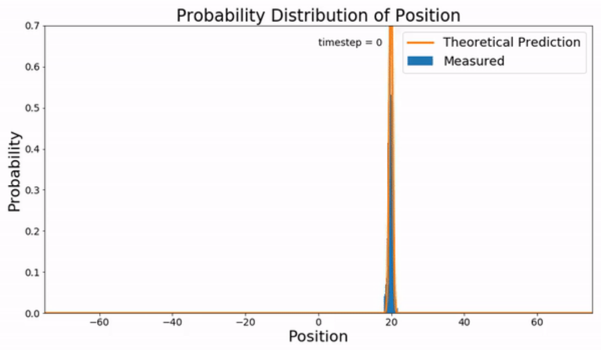


Figure: Solution to a 1D Fokker-Planck equation.

Rotation Statistics: Diffusion Approximation

Why It's Hard for Spinning Dust Grains

At the timescale of the internal processes:

$$\frac{\partial f(\ell,m)}{\partial t} = \sum_{\ell' m'} \Big\{ w_{\mathrm{p}}(\ell', m' \to \ell, m) f(\ell', m') - w_{\mathrm{p}}(\ell, m \to \ell', m') f(\ell, m) \Big\} + \sum_{m'} \Big\{ w_{\mathrm{d}}(m' \to m) f(\ell, m') - w_{\mathrm{d}}(m \to m') f(\ell, m) \Big\},$$

and $\partial_t f(\ell) = \sum_m \partial_t f(\ell, m)$.

Non-equilibrium system

- The internal processes could have a much higher statistical temperature.
- When deriving drift or fluctuation rates for certain processes, quasi-equilibrium approaches - such as directly applying the detailed balance rule - may fail.

CONCLUSION

```python
from SpyDust.SpyDust import SpyDust
params = {"nh" : 30, "T": 100., "Chi": 1, "xh": 1.2e-3, "xC": 3e-4, "y" : 0, "gamma": 0, "dipole": 9.3,
  "line":7}
spectrum = SpyDust(params, min_freq=1, max_freq=400, n_freq=200, single_beta=True)

```

- SpyDust is an improved and extended *Python* implementation for modelling spinning dust emission.
- Support for polarisation (full Stokes) and moment expansion is in development — stay tuned!
- Most dimensions of the problem can be reduced for SED representation.
- The rotation distribution remains the irreducible core — it cannot be abstracted away.